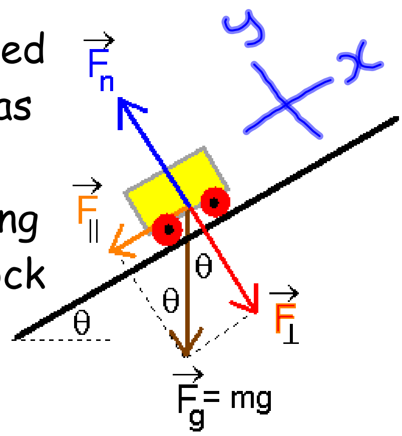
# Banked Curves

When a car travels along a horizontal curve, the centripetal force is usually provided by the force of friction between the car's tires and the road's surface. To reduce the reliance on friction we can incline or bank the curve relative to the horizontal. This method of banking curves is used with race cars on circular or oval tracks and on highway on- and off-ramps.

## Difference Between Incline Planes and Banked Curves

In the section on inclined planes, you learned how to deal with the forces on an object as it went up (or down) an incline Actually, most of that lesson was about blocks sliding on a plane, but if we put wheels on the block then it becomes a car!

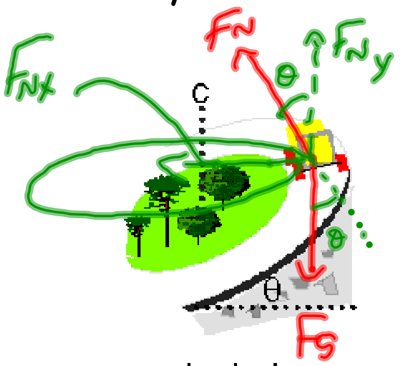



The "car" in the picture will definitely move down the hill. First, there is no friction, second it has wheels and third, the gears are in neutral! You might say that the main force acting on the car is its weight, **mg**.

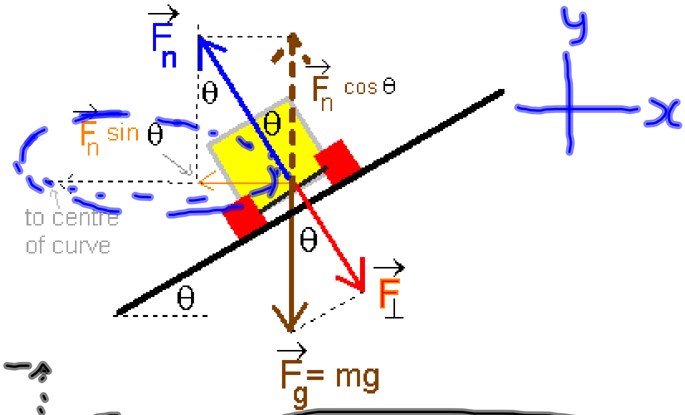
The weight vector is resolved into two components, one perpendicular to the hill and one parallel to the hill. The parallel force makes the car move down the hill.

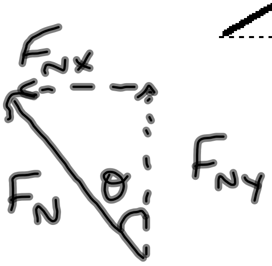
The force which is perpendicular causes the hill to exert a force back on the car. This latter force is called the normal force, Fn. You should be able to see that **Fn < Fg**. This is not always the case as you will see when a car goes around a banked curve.

Next we put the car on a banked curve and then look directly at it "end-on" and draw a free-body diagram.




car on banked curve with centre C




$$\boxed{F_{Nx} = F_N \sin\theta}$$

$$F_{Ny} = F_N \cos\theta$$

Look at the first picture where the car is rolling down the incline. Notice that the "most important" force was the weight **mg**.

Now look at the car on the banked curve. It's not rolling down the hill, and the "most important" force here is **Fn**.

You can see that **Fn** is resolved into an vector that points towards the centre of the banked curve , and a dashed vertical vector.

The solid vector is Fn sin θ and the dashed vector is Fn cos θ. Since we stipulated that there is no friction, Fn sin θ is a very important force. Without it the car would slip off the banked curve and into the ditch.

In other words, Fn sin θ is the centripetal force directed towards the centre of the curve (not parallel with the bank!)

Since Fn sin θ is the centripetal force,

$$F_N \sin\theta = \frac{mv^2}{r} \qquad \text{Equation 1}$$

Look at the free-body diagram again to see that

$$F_N \cos\theta = mg \qquad \text{Equation 2}$$

Solving the second equation for Fn yeilds

$$F_N = \frac{mg}{\cos\theta} \qquad \text{Equation 3}$$

Substitute equation 3 into equation 1:

$$F_n \sin\theta = \frac{mv^2}{r}$$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{mv^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$\boxed{r = \frac{v^2}{g\tan\theta}}$$

given

The last line in the above computation gives the radius of a curve that safely allows a car to manoeuver around it at speed v when there is no friction (for example, in a black ice situation).

As you can see from the above formula, the radius is directly proportional to the velocity squared for a curve banked at an angle θ.

Note that the line can be rearranged to give v as the subject:

$$v = \sqrt{gr \tan\theta}$$

For a given banked curve, there is only one speed at which the centripetal force is provided strictly by a component of the normal force. At this speed, the object does not require a frictional force to undergo uniform circular motion. If the car moves faster than this speed, it will slip up the banked curve, and if it goes slower than this speed, it will slip down the bank (if the road is perfectly frictionless).

Notice that mass is missing form the expressions.  This means that both heavy and light vehicles behave identically on banked curves.

## Examples

1. A race car travels along a banked curve at a speed of 120 km/h. It does not depend on the force of friction to keep it on the track. If the turn is banked at an angle of 25° to the horizontal, what is the radius of rotation?

$V = 120 km/h$

$V = 33.3 m/s$

$\theta = 25°$

$r =$

$$r = \frac{V^2}{g \tan\theta}$$

$$r = \frac{(33.3 m/s)^2}{9.8 m/s^2 \tan 25°}$$

$r = 240m$

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2. Because landfill is expensive, a race track owner decides to bank curves at no greater angle that 22°. If a driver is to safely steer around a curve at 110 km/h under frictionless conditions, what must be the radius of the bank curve?

$$r = \frac{v^2}{g\tan\theta} = \frac{(30.6\,m/s)^2}{(9.8\,m/s^2)\tan 22^\circ} = 240\,m$$

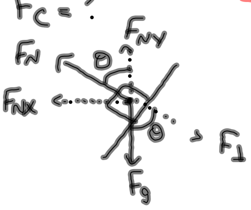
3. A 1200 kg car is rounding a frictionless curve banked at an angle of 27°.
   A) What is the centripetal force acting on the car?
   B) What speed should the car round the curve, without slipping, if the radius of the curve is 190m?

A)

$m = 1200\,kg$

$\theta = 27^\circ$

$F_c = ?$




① $F_{Ny} = F_g$ ② $F_c = F_{Nx}$

method 1:

① $F_{Ny} = F_g$

$F_N\cos\theta = mg$

$F_N = \frac{mg}{\cos\theta}$

$F_N = \frac{(1200\,kg)(9.8\,m/s^2)}{\cos 27^\circ}$

$F_N = 13\,200\,N$

② $F_c = F_{Nx}$

$F_c = F_N\sin\theta$

$= 13200\,N \sin 27^\circ$

$= 6.0\times10^3\,N$

or method 2

① $F_{Ny} = F_g$

$\frac{F_N\cos\theta}{\cos\theta} = \frac{mg}{\cos\theta}$

$F_N = \frac{mg}{\cos\theta}$

② $F_c = F_{Nx}$

$F_c = F_N\sin\theta$

$F_c = \frac{mg}{\cos\theta}\cdot\sin\theta$

$\boxed{F_c = mg\tan\theta}$

(b) $F_c = \frac{mv^2}{r}$

$V = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(6000\,N)(190\,m)}{1200\,kg}}$

$V = 31\,m/s$

$V = \sqrt{gr\tan\theta}$

Do questions #4 p. 214 and # 19 p. 226